PROJECTIVE GEOMETRY.

Premiers Principes de Géométrie Moderne. Par E. Duporcq. Pp. viii + 160. (Paris: Gauthier-Villars, 1899.)

IT is a curious fact that while projective geometry is becoming better appreciated in England it seems to be going out of favour in France. M. Duporcq, in his introduction, pathetically deplores the predominant place assigned to analysis in the syllabuses of the official examinations; and in France, as with ourselves, most teachers are compelled to neglect a subject that does not pay. It will be sad indeed if, in the fatherland of Monge, Poncelet and Chasles, pure geometry is to be deposed from her former high estate, and made a kind of Cinderella, called in to do odd jobs for Her Serene Highness the Princess *Analyse*, or to amuse the children with tricks of the triangle.

M. Duporcq's book itself helps us to realise the danger that is threatened. One cannot help feeling that his attitude is apologetic, and that his exposition is a half-hearted one. At the very outset we are confronted with homogeneous coordinates; homography is based on an algebraic relation; points at infinity lie in a plane "by definition"; imaginary elements have no real existence, and the introduction of them, due to analysis, is a mere *façon de parler*, vaguely justified by the "Principle of Continuity." With all respect to Poncelet, it may be doubted whether his "principle of continuity," apart from algebraical considerations, has any real working value; on the other hand, von Staudt elaborated, forty years ago, a theory of imaginary elements which, so far as curves and surfaces of the second order are concerned, gives a consistent geometrical theory (quite independent of analysis) in which the principle of continuity has a real meaning, and is at the same time practically self-evident, as one would expect it to be. Von Staudt's name does not appear to be mentioned in M. Duporcq's book, and the reader might not unreasonably infer that the author was ignorant of v. Staudt's existence.

It would, of course, be absurd to advocate the exclusive use of pure, as opposed to analytical, geometry, even in problems of a strictly geometrical character. The ideal geometrician should be equally expert in both methods, and apply one or the other or both combined according as circumstances may require. But it may fairly be urged that a treatise on the *first principles* of projective geometry should avoid the introduction of coordinates except by way of illustration, and for the purpose of showing the points of contact between the two methods. It is right to teach an apprentice the use of a saw as well as that of a plane; but you will not attain this end by giving him a tool that is neither a saw nor a plane, but contains something of both.

Thus to give an explicit example, M. Duporcq frequently infers homography from a one-to-one relation established, not from an equation, but from the inspection of a figure. Thus (p. 49):

"Si donc m et m' désignent les deux points où une droite quelconque Δ coupe une conique circonscrite au quadrangle $a\,b\,c\,d$, on voit qu'à tout point m de Δ ne correspond ainsi qu'un point m'. Comme, d'ailleurs, ces points sont évidemment réciproques, ils déterminent donc une involution sur Δ," &c.

These statements are doubtless correct, but are they sufficiently justified? How is the beginner to distinguish the argument from the following:

"Two points S, H are taken on a tangent to an ellipse, and any ellipse with foci S, H cuts the given ellipse in the points M, M': then to each point M corresponds one point M' and *vice versâ*, hence we have a system in involution, and MM' goes through a fixed point"?

It is not a sufficient answer to say that M, M' are only a pair of four associated points, because this is not geometrically evident. Again, we have cases of Cremona correspondence with the fixed points imaginary: how is the untrained student to distinguish them from homographic correspondences?

We are far from wishing to suggest that M. Duporcq's work is devoid of interest and value. Considering its size it is remarkable for the range and variety of its contents; it comprises a very attractive and, indeed, brilliant sketch of homography, poles and polars, involution, quadratic transformation (including inversion), together with an outline of Lie's line-sphere correspondence. For a reader prepared by previous study, it affords an excellent and suggestive *résumé*; it is rather when it is examined as a methodical text-book for students that it seems to us to fall short of perfection. To the student we would still say: Read Reye, work his exercises, and then, if you like the subject, gird up your loins and tackle von Staudt. For it is a truth past gainsaying that v. Staudt's "Geometrie der Lage" and the immortal "Beiträge" contain, as no other books do, the essentials of projective geometry. G. B. M.

A SYSTEM OF PHYSICS.

Kanon der Physik. By Felix Auerbach. Pp. xii + 522. (Leipzig: Viet and Co., 1899.)

SCIENTIFIC books may be divided into two groups, those which are written because the author has something to teach, and those which are written because he has something to learn. It is no reproach to a writer if his book is classed with the second group, for there may be as much originality in learning as in teaching, and his autodidactic efforts will often prove a source of instruction to others. It is not possible to say whether Prof. Auerbach has been consciously writing his "Kanon" of physics to clear up his own ideas on scientific principles, but the book he has produced gives the impression that this has been one of his principal motives; and I would even go a step further and say that, if life were long enough, every physicist ought, when he gets to the age of fifty, to spend three years in putting his ideas into shape and write a similar treatise. It would serve as a kind of "Abiturienten Examen" to his state of crystallisation.

It is easier to talk about this book in vague and general terms than to give an account of what it is and what it contains. I am afraid of becoming definite in my own words, for fear of giving a wrong impression, and must content myself with the translation of a few sentences taken out of the preface.

"A comprehensive book is still wanting—and not only in Germany—in which the conceptions, principles,

theorems and formulæ, dimensional relations and numbers belonging to physics are represented and put together in a systematic manner, and in a way which would do justice to two different intentions: to give on the one hand to the reader a general view of the whole, without disturbing him by methodical, historical and other details, and on the other hand to give without circumlocution, to any one who may consult the book, a definite answer to his questions. The great difficulty in principle of such a 'Kanon' of physics lies clearly in the fact that it is in exact science often, perhaps generally, impossible to give a short answer to a short question. If a scientific man is asked, What is mass? What is elasticity? What is entropy? he takes a long breath and begins with a long introduction—and not without reason; for the difficulty of a short and detached answer lies in the nature of the thing. But we must not consider the difficulty to be insurmountable. We must fix our mind on what is essential and characteristic, and give expression to it in our answer; as for the rest, so far as is necessary, it may be added afterwards by supplementary remarks."

The volume which the author has produced to satisfy his requirements is worth reading because it is stimulating. Irritating would perhaps be the better word for the principal sensation felt in perusing it, because we constantly come across statements which do not seem to coincide with our own views, or with explanations which do not satisfy; yet in spite of trying hard, it is difficult sometimes to point out what it is that does not satisfy, and even if one succeeds one feels that the thought one has been obliged to give to the matter has cleared and perhaps modified one's own views. The book begins with a number of chapters on general principles, space, time, motion, force and mass and the properties of matter. The subject is treated in a concise, short and instructive manner, but the author does not always succeed in giving us, as promised in the above passage, a short answer to a short question. His definition of dispersion, *e.g.*, takes up eight lines and wants reading eight times before it can be understood. After a short chapter in which the principal equations of the potential theory are put together and explained, the author enters into the two chief divisions of the book, "energy" and "entropy." He takes a rather wide and unusual view of the latter word, including under it all transformations of energy. Without entering into the difficult question of classification, we may commend these two chapters, which most physicists will read with profit. But surely a better definition of electric current might be given than the one on p. 250:—

"When one observes, that the potential has different values at different points of a conductor, one expresses this fact also by saying: a certain quantity of electricity moves in the conductor, or an electric current flows in it."

The difference of potential at different points is by no means characteristic of an electric current, as for instance in air, where we are constantly dealing with such differences. Dr. Auerbach, to make his explanation correct, must therefore lay stress on a sharp distinction between conductors and non-conductors; but how would he define a conductor except by arguing in a circle, and saying that a body is a conductor when a fall of potential causes an electric current.

Among the points of the book which are irritating without being stimulating, we may mention the very annoying method of numbering concurrently and independently his paragraphs, according as they contain matters of principle or laws and propositions. Thus § 52, printed in fat type, follows § 158 printed in somewhat leaner characters, and whenever a reference has to be looked up, one has to investigate the type carefully, and if it is, *e.g.*, 91 fat, turn to p. 385; while if it is 91 lean, find the required passage on p. 118. We hope that in future editions a different system will be adopted.

Lecturers will find one use, perhaps not a very high one, for this book; it will save them thought and labour, by helping them to arrange their course in a systematic and orderly fashion. ARTHUR SCHUSTER.

OUR BOOK SHELF.

Insects: their Structure and Life. A Primer of Entomology. By George H. Carpenter, B.Sc. (Lond.). Pp. xi + 404. (London: J. M. Dent and Co., 1899.)

MR. CARPENTER, Assistant Naturalist in the Science and Art Museum in Dublin, is favourably known to entomologists by numerous valuable papers on *Lepidoptera*, *Odonata*, cave-insects, economic entomology, &c.; and we are very pleased to welcome a useful introductory manual of entomology from his pen. It is compiled from a variety of sources, special use having been made, in the chapters on the form and life-history of insects, of the well-known work on the cockroach by Profs. Miall and Denny. These chapters will be found very useful, especially as the names attached to the various parts of insects are clearly and carefully explained. Classification and the principal orders and families of insects are then dealt with as fully as the space at the author's disposal would allow; and chapters on insects and their surroundings and on the pedigree of insects close the body of the book, which concludes with a short bibliography and a good index. Perhaps Chapter v., on insects and their surroundings, will be found most interesting to the general reader; for it treats of such subjects as cave-insects, fresh-water insects, marine insects, geographical distribution, mimicry, &c. Mr. Carpenter usually expresses himself very cautiously, but when he says that the number of described species of insects amounts to a quarter of a million, and that there are probably two millions of species still undescribed, we are inclined to think that both his estimates are very much below the mark. The number of described species of insects cannot be less than 300,000 at present, and many entomologists think that the late Prof. Riley's estimate of the number of existing species of insects as ten millions is by no means to be regarded as extravagant. Mr. Carpenter's remarks on the various subjects connected with evolution are very well expressed and reasoned out. W. F. K.

Year-book of the United States Department of Agriculture, 1898. Pp. 768. (Washington: Government Printing Office, 1899.)

THE volume before us, like so many of its predecessors which have been noticed in these columns, is full of contributions of interest and value to students of science, agriculturists and others. Although appealing primarily to residents in the States, many useful hints and suggestions may be gleaned from the year-book by its readers in this country. The report of the Secretary of the Department shows that the varied operations carried on have been prosecuted with vigour. The Department has at present four scientific explorers abroad getting seeds and plants—one in Russia, one in the countries